

Lens Theory

Singlet Lenses - Nomenclature

Below is a common diagram used to define quantities for the analysis of a singlet lens. The quantities defined can also be extended to multi-element systems by procedures outlined in optical design textbooks.

The optical axis ($O-O'$) of the lens is a line passing through the centers of curvature of the two spherical lens surfaces (centers of curvature not shown).

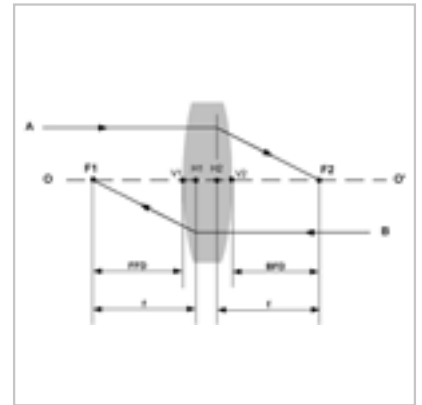
Ray **A** is incident from **left to right** and parallel to the optical axis. Ray **A** is refracted to point **F2** on axis, the **back, second, or secondary focal point**.

Ray **B** is incident from **right to left** and parallel to the optical axis. Ray **B** is refracted to point **F1** on axis, the **front, first, or primary focal point**.

Suppose the lens is „well-corrected“. That means all rays **A** parallel to the axis are focused to a single point **F2**, regardless of their distance from the axis. It is clear then that the family of rays refracted to **F2** intersect the family of progenitor rays **A** in a sphere centered at **F2**. This sphere, or, in general, surface of intersection of progenitor and refracted rays is called the **back, second, or secondary principal surface**. The intersection **H2** of this surface with the optical axis is called the **back, second, or secondary principal point**.

Similarly, rays **B** incident from the right, and their refracted rays impinging on **F1**, intersect to form a **front, first, or primary principal surface**, which intersects the optical axis at **H1**, the **front, first, or primary principal point**.

In the paraxial approximation, all ray angles are small. This approximation permits the modelling of the principal surfaces as **principal planes**. The distances from the front and back principal points to their respective focal points are equal and given by **f**, the **focal length** or **effective focal length**. The **front and back focal distances FFD and BFD**, respectively, are measured from the surface intersections **V1** and **V2** to the respective foci. **V1** and **V2** are called the front and back vertices.



Singlet Lenses - Quantitative

To calculate these quantities above we need sign conventions.

1. Distance increases towards the right.
2. A surface has positive curvature if its center of curvature is to the right of the vertex of the surface. In other words, if you have to move in the positive direction from the vertex of a surface to reach its center of curvature, the curvature is positive.
3. Angles increase from zero, parallel to the optical axis, to positive values in a counterclockwise direction from the optical axis.
4. V_1H_1 and H_2V_2 are positive if the principal points are to the right and left, respectively, of their corresponding vertices. Note that the principal points can fall outside of the lens.





The formulas below now permit the calculation of the effective focal length, front and back focal distances, and positions of the principal points of a singlet lens in the paraxial approximation. The distance t_c below is the center thickness of the lens V_1V_2 .

Important Quantities for Singlet Lenses Immersed in Air.

Symbol	Description	Formular
f	effective focal length	$\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] + \frac{t_c (n-1)^2}{nR_1R_2}$
BFD	back focal distance	$BFD = f \left[1 - \frac{t_c (n-1)}{nR_1} \right]$
FFD	front focal distance	$FFD = f \left[1 + \frac{t_c (n-1)}{nR_2} \right]$
H_2V_2	back principal point to back vertex distance	$H_2V_2 = f - BFD = f \frac{t_c (n-1)}{nR_1}$
V_1H_1	front vertex to front principal point distance	$V_1H_1 = f - FFD = -f \frac{t_c (n-1)}{nR_2}$

For plano convex, plano concave, equiconvex and equiconcave lenses, we give the focal lengths in the following table. (Principal plane data may be calculated using the tables above.)

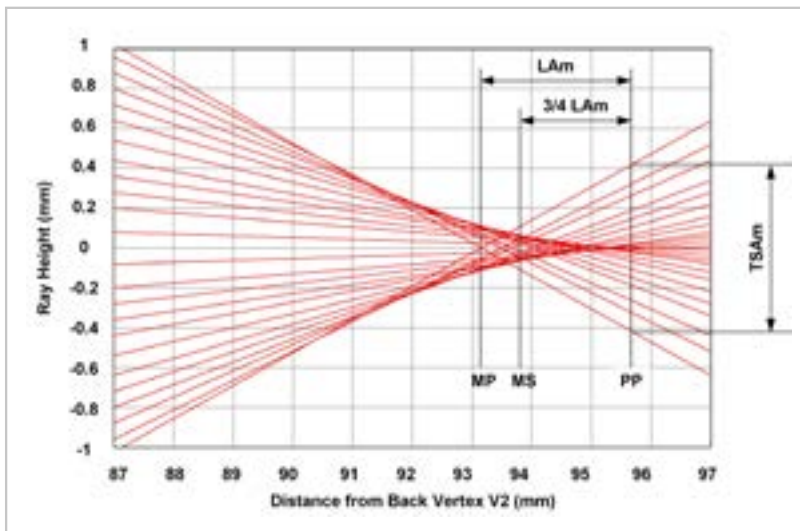
Focal Length Formulas for Simple Singlet Lenses in Air.
The Radii are considered positive in the formulas below.

Type	Description	Formular
Plano Convex		$f = \frac{R}{(n-1)}$
Plano Concave		$f = -\frac{R}{(n-1)}$
Equiconvex		$f = \left[\frac{2(n-1)}{R} - \frac{t_c(n-1)^2}{nR^2} \right]^{-1}$
Equiconcave		$f = \left[\frac{2(n-1)}{R} + \frac{t_c(n-1)^2}{nR^2} \right]^{-1}$

In the singlet lens tables in our catalog, focal lengths at a few key wavelengths are given. We also give very accurate radii of curvature and center thickness data for each lens. Use the formulas in the above table and the values for n given in the index of refraction tables of our catalog.

Performance and Proper Orientation of Singlet Lenses for Minimum Spot Size

High quality singlet lenses are of particular interest in laser focusing and beam handling applications because of their low cost, high damage threshold, and availability of a large variety of standard parts. We offer a large selection of BK7 and UV grade fused silica singlets that can satisfy most monochromatic focusing requirements. Can singlet lenses provide diffraction limited performance in the focusing of collimated beams? Examine the figure below.



This figure is a geometrical ray trace of 25 rays in the focal region of a 100 mm focal length BK7 plano convex lens ($n=1.515$). The lens is oriented with the plano surface towards the focus and the origin (0 mm) is at the back vertex (V2) of the lens. The rays were launched parallel to the axis and equally spaced in a region 16 mm above and below the axis in a plane containing the axis (**meridional plane**). Therefore, the lens is operating at the rather "fast" aperture of $f/3.125$. The **marginal rays** reach the axis first, in plane **MP**. The **paraxial rays** reach the axis last, at the **paraxial focal plane PP**. The distance from **V2** to **PP** is the back focal distance (**BFD**) of the 100 mm focal length lens. This can be verified using the formulas previously defined. The distance from **PP** to **MP**, here negative, is the **longitudinal spherical aberration LAm** of the marginal rays. The beam width in the paraxial focal plane **PP** is the **transverse spherical aberration TSAm**, determined by the heights of the marginal rays. Notice that the smallest geometrical spot size can be found at plane **MS**, approximately $3/4$ LAm back toward the lens from the paraxial focal plane.

Shape Factor

For singlet lenses, the smallest beam diameter d_{MS} at plane **MS** can be computed from third order aberration theory. The result is:

$$d_{MS} = f \Delta\theta \text{ blur}$$

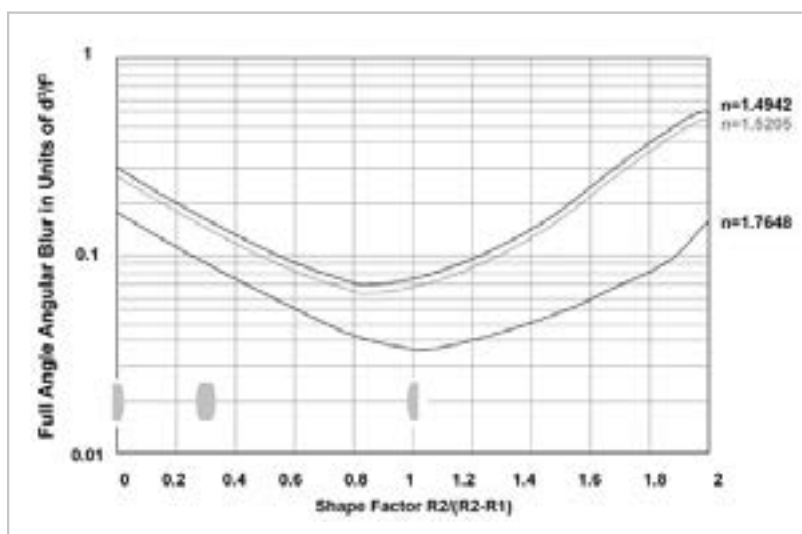
where $\Delta\theta \text{ blur}$ is the "full angle angular blur" of the lens for collimated input light, given by:

$$\Delta\theta \text{ blur} = (d_0/f)^3 [n^2 - (2n+1)K + (n+2)K^2 / n] / 32(n-1)^2$$

where d_0 is the input beam diameter and K is the shape factor of the lens given by:

$$K = R_2 / (R_2 - R_1)$$

Note that the lens orientation is contained in K . The angular blur is plotted as a function of shape factor for three typical glasses used in our singlets.



Geometrical optics angular spot size for spherical singlet lens for indices of refraction typical of fused silica ($n = 1.4942$ at 280.0 nm), BK7 ($n = 1.5205$ at 514.5 nm) and SF11 ($n = 1.7648$ at 800.0 nm). A collimated beam is assumed incident on R1. To use this graph, find the shape factor of the lens using the proper sign conventions for R1 and R2. Read the angular blur, in units where d is the beam diameter and f is the focal length. Now calculate the actual angular blur in radians by multiplying by $(d0/f)^3$. Finally, multiply by the focal length to obtain the spot size in units of length. If the value is much less than the diffraction limit, performance of the lens will be limited by diffraction rather than spherical aberration.

Three important points should be noted:

1. The error of misorienting a plano convex lens approximately quadruples the blur.
2. Use of a best form lens (shape factor corresponding to the blur minima) for any of the three glasses shown is hardly worth the expense.
3. The high index of SF11 significantly reduces singlet aberration.

For this example, ($K = 1$), we compute a geometrical optics minimum beam size $d_{MS} = 226$ μm . L_{Am} is given by:

$$L_{Am} = -4f^2 \Delta\theta_{\text{blur}} / d0$$

The minimum spot size occurs at a distance $s_2 = \text{BFD} - 3/4 L_{Am}$ from the rear vertex of the lens. In this example, $L_{Am} = -2.82$ mm and $s_2 = 93.92$ mm.

A 226 μm spot size obtained by focusing a 32 mm diameter collimated beam is hardly diffraction limited. We have purposely chosen a highly aberrated situation as an example. The diffraction limited full angle angular blur with aperture limited by the beam diameter $d0$ is:

$$\Delta\theta_{\text{diff}} = 2.44 \lambda / d0$$

implying a diffraction limited spot size of:

$$d_{\text{diff}} = f \Delta\theta_{\text{diff}} = 2.44 \lambda f / d0$$

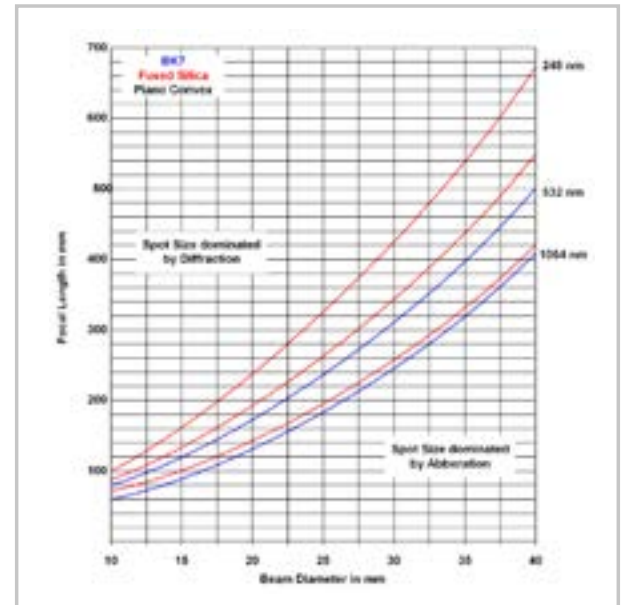
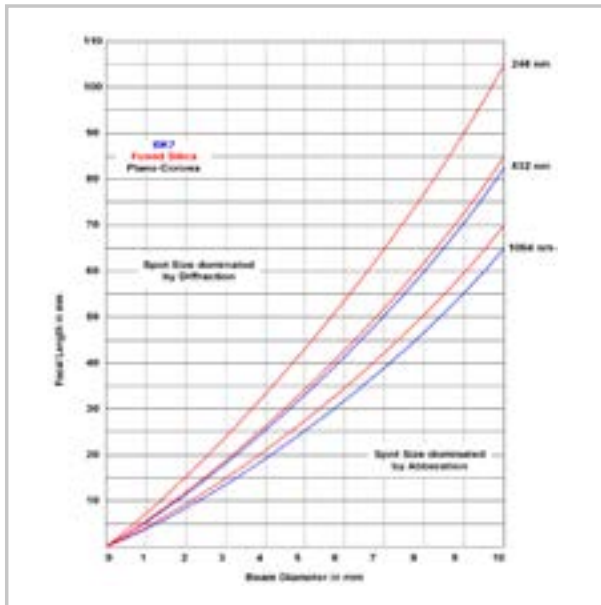
This is the diameter of the first dark ring of the Airy pattern in the focal plane. For the above example, $d_{\text{diff}} = 4.8$ μm at a wavelength of 632.8 nm. Comparing the aberration limited geometrical optic spot size $f \Delta\theta_{\text{blur}}$ to the diffraction limited spot size $f \Delta\theta_{\text{diff}}$ suggests a criterion for diffraction limited operation of a singlet lens. Compute the focal length as a function beam diameter where

$$f \Delta\theta_{\text{blur}} = f \Delta\theta_{\text{diff}}$$

If $f \Delta\theta_{\text{blur}} > f \Delta\theta_{\text{diff}}$ the spot size is limited by aberration.

If $f \Delta\theta_{\text{blur}} < f \Delta\theta_{\text{diff}}$ the spot size is essentially diffraction limited.

The curves on the following page depict this computation for properly oriented plano convex BK7 and fused silica singlets. To use the curves, find the beam diameter and desired focal length point on the graph. If this point is above the curve for intended wavelength, plano convex singlet lens performance is essentially diffraction limited. If this point is on or below curve, consider using a doublet to decrease aberrated spot size.



Additional Notes

1. In designing optical systems for low divergence laser beams, the paraxial approximation is excellent as far as the positioning of the elements is concerned. However, due to diffraction, the magnifications encountered may be completely different than those calculated from geometric optics. Use the familiar ABCD matrix formalism. If you treat the lenses as thin lenses, use distances between principal points to determine the element positioning. If you use the actual radii of curvature, indices of refraction, and center thicknesses in a more detailed calculation, your calculation will already implicitly take into account the principal planes.
2. Formulas for Gaussian beam waists are readily derived using standard ABCD matrix formalism. Do not take the beam diameter results too seriously, however, unless you are working with cw HeNe, ion, Ti:Sapphire, Nd:YAG, or dye lasers with published and verifiable TEM₀₀ waists and locations. In particular, "near-Gaussian" pulsed solid state lasers are many times diffraction limited as may be verified by comparing the manufacturers beam diameter and full angle divergence specifications. Single-stage excimer and copper vapor lasers are even more divergent, even when using unstable resonator optics. (Here, the beam divergence actually varies during the pulse, with the brightness increasing with round trip number.) For these lasers, the divergence limited spot size is given by

$$d^2 \cdot \Delta\theta,$$

where $\Delta\theta$ is the **measured** full angle divergence. It is this spot size that should be computed and compared to the spot size due to spherical aberration in the choice of a focusing lens.

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