

Basic Principles of Optics

Diagram Nomenclature

The adjoining drawing is used to define terms and quantities of singlet lenses.

The optical axis (OO') of the lens is the line passing through the centers of curvature of the two spherical lens surfaces. The centers of curvature are not shown in the diagram.

Ray A runs parallel to the optical axis and is refracted to point F_2 . F_2 is called the back, secondary, or image focal point.

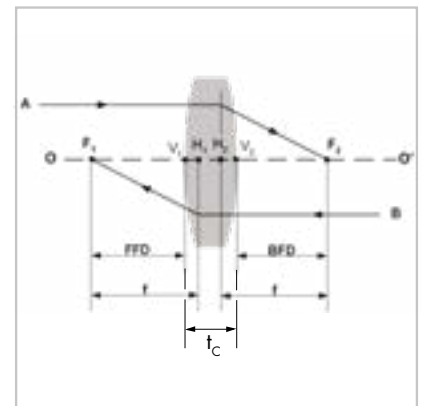
Ray B also runs parallel to the optical axis but in the opposite direction. It is refracted to point F_1 , the so-called front, primary, or object focal point.

"Well corrected" lenses focus all rays parallel to the optical axis to a single focal point.

Lenses in which the thickness cannot be neglected can be viewed as optical systems consisting of multiple lenses. The focal points F_1 and F_2 are measured from the vertices V_1 and V_2 . The distances FFD and BFD are called front and back focal distance, respectively.

The intersecting points of the extended progenitor and refracted rays form a sphere. For the area close to the optical axis this sphere can be approximated by a plane, which is known as principal surface. The intersections H_1 and H_2 of the principal surface with the optical axis are called the front and back principal point, respectively. The distances from the principal points to their respective focal points (H_1F_1 and H_2F_2) are represented by f , the effective focal length.

For common glass lenses the distance between the principal planes is approximately one third of the lens thickness $V_1V_2 = t_c$.



Sign Conventions

To calculate the described dimensions we need the following sign conventions:

- Distance increases towards the right.
- A surface has positive curvature if its center of curvature is to the right of the vertex of the surface.
- Angles increase from zero, parallel to the optical axis, to positive values in a counter-clockwise direction from the optical axis.
- V_1H_1 and H_2V_2 are positive if the principal points are to the right and left, respectively, of their corresponding vertices. Note that the principal points can fall outside of the lens.





Collection of Formulas for Lens Calculation

The following formulas are used to calculate the different specifications. R_1 denotes the radius of curvature of the incident side, R_2 the radius of curvature of the output side.

General Formulas for Lenses Immersed in Air

| Symbol | Description | Formula |
|----------------------|-----------------------------------|--|
| f | Effective focal length | $\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} \right] + \frac{t_c (n-1)^2}{nR_1R_2}$ |
| BFD | Back focal length | $\text{BFD} = f \left[1 - \frac{t_c (n-1)}{nR_1} \right]$ |
| FFD | Front focal distance | $\text{FFD} = f \left[1 + \frac{t_c (n-1)}{nR_2} \right]$ |
| H_2V_2 V_1H_1 | Distance principal point - vertex | $H_2V_2 = f - \text{BFD} = f \frac{t_c (n-1)}{nR_1}$ $V_1H_1 = f - \text{FFD} = -f \frac{t_c (n-1)}{nR_2}$ |

Selected Cases of the Focal Distances for Thin Lenses Immersed in Air

| Type | Description | Formula |
|---------------|--|--|
| Plano-convex |  | $f = \frac{R}{(n-1)}$ |
| Plano-concave |  | $f = -\frac{R}{(n-1)}$ |
| Biconvex |  | $f = \left[\frac{2(n-1)}{R} - \frac{t_c (n-1)^2}{nR^2} \right]^{-1}$ |
| Biconcave |  | $f = -\left[\frac{2(n-1)}{R} + \frac{t_c (n-1)^2}{nR^2} \right]^{-1}$ |

In the singlet overview of our [laser optics catalog](#), you will find focal length values for the most important wavelengths. The exact radii of curvature and center thickness values are also stated. You can use the listed formulas to calculate additional focal lengths. Specifications regarding the refraction indices of the different substrates can be found from page 28 on in the refraction index tables.

Determination of the Spherical Aberration

High quality singlet lenses are of particular interest in laser focusing and beam handling applications because of their low cost, high damage threshold, and availability of a large variety of standard parts.

LASER COMPONENTS offers a large selection of BK7 and UV grade quartz singlets that can satisfy most monochromatic application requirements.

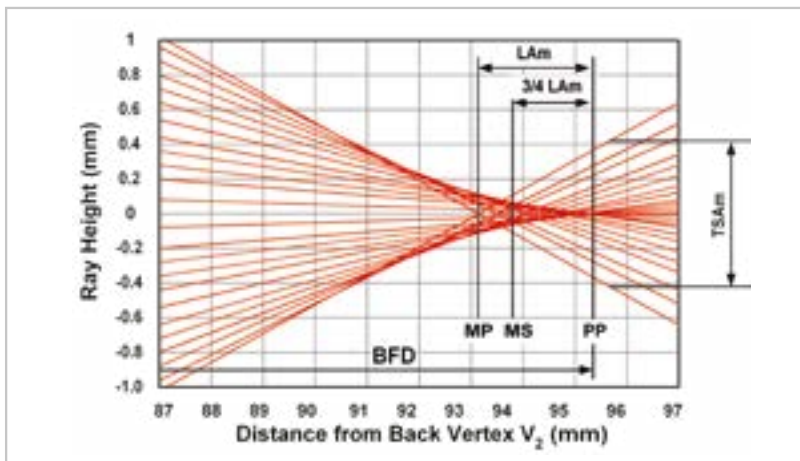
Only in the Gaussian area do spherical surfaces deliver ideal images. It is therefore recommendable to simulate how high the deviations outside the area are in order to determine possible applications.

Simulation

This simulation is a geometrical ray trace of 25 rays in the focal region of a 100 mm focal length BK7 plano-convex lens ($n = 1.515$).

A collimated laser beam enters the convex side of the lens and exits on the plane side. The simulated rays are launched parallel to the optical axis and are equally spaced in a region 16 mm above and below the axis.

The origin of the graph (0 mm) is at the back vertex (V_2) of the lens. The x-axis hence shows the back focal distance (BFD).



Simulation Result

- The marginal rays intersect the optical axis in plane MP while the paraxial rays reach the axis at focal plane PP.
- The distance V_2PP is the back focal distance (BFD) of the lens.
- The distance $PPMP$, here negative, is the longitudinal spherical aberration LAm of the marginal rays.
- The beam width in the paraxial focal plane PP is the transverse spherical aberration $TSAm$, determined by the height of the marginal rays (and hence by the diameter of the incident beam).

Notice that the smallest geometrical spot size can be found at plane MS, approximately $3/4 LAm$ back toward the lens from the paraxial focal plane PP.

Shape Factor

For singlet lenses, the smallest beam diameter d_{MS} at plane MS can be computed from the third order aberration theory. The result is:

$$d_{MS} = f \Delta\theta_{blur}$$

where $\Delta\theta_{blur}$ is the "full angle angular blur" of the lens for collimated input light, given by:

$$\Delta\theta_{blur} = (d_0/f)^3 [n^2 - (2n+1)K + (n+2)K^2/n] / 32(n-1)^2$$

where d_0 : Input beam diameter

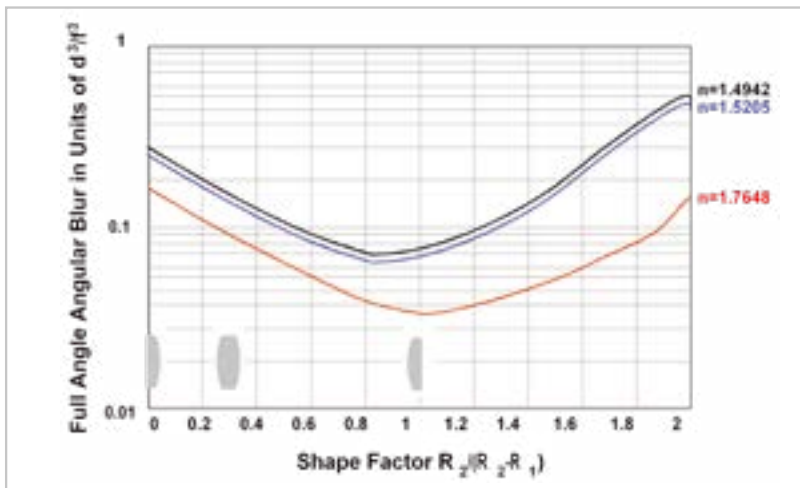
K: Shape factor of the lens given by: $K = R_2 / (R_2 - R_1)$

Note that the lens orientation is contained in K.

Angular Blur

Assuming geometrical optics conditions, the graph depicts the angular blur as a function of the shape factor of a spherical singlet lens for typical refractive indices:

- Quartz ($n = 1.4942$ at 280.0 nm)
- BK7 ($n = 1.5205$ at 514.5 nm)
- SF11 ($n = 1.7648$ at 800.0 nm)



Determination of the Influence of the Spherical Aberration

A collimated beam is assumed incident on the lens surface with radius R_1 . To use this graph, find the shape factor of the lens using the proper sign conventions for R_1 and R_2 . Read the angular blur, in units where d is the beam diameter and f is the focal length. Then calculate the actual angular blur in radians by multiplying by $(d_0/f)^3$. Finally, multiply by the focal length to obtain the spot size in units of length.

If the value is much less than the diffraction limit, performance of the lens will be limited by diffraction rather than spherical aberration.

Important to Note:

- The collimated laser beam should first hit the curved surface. The error of misorienting a plano-convex lens approximately quadruples the blur.
- Use of a best form lens is hardly worth the expense.
- The high index of SF11 ($n = 1.799$ at 515 nm) significantly reduces singlet aberration.

Calculation Example

For $K = 1$ we can compute the geometrical optics minimum beam size of $d_{MS} = 226 \mu\text{m}$.

LAm is given by: $LAm = -4f \Delta\theta_{\text{blur}}/d_0$

The minimum spot size occurs at a distance of

$s_2 = \text{BFD} - 0.75 LAm$ from the rear vertex of the lens. In this example, $LAm = -2.82 \text{ mm}$ and $s_2 = 93.92 \text{ mm}$.

A $226 \mu\text{m}$ spot size obtained by focusing a 32 mm diameter collimated beam possesses a strong aberration.

The diffraction limited full angle angular blur with aperture limited by the beam diameter d_0 is:

$$\Delta\theta_{\text{diff}} = 2.44 \lambda/d_0$$

Implying a diffraction limited spot size of:

$$d_{\text{diff}} = f \Delta\theta_{\text{diff}} = 2.44 \lambda f / d_0$$

d_{diff} is the diameter of the first dark ring of the Airy pattern in the focal plane.

For the above example, $d_{\text{diff}} = 4.8 \mu\text{m}$ at a wavelength of 632.8 nm .

The diffraction limit is a function of focal distance f and the beam diameter. The depicted graph is computed by

$$f \Delta\theta_{\text{blur}} = f \Delta\theta_{\text{diff}}$$

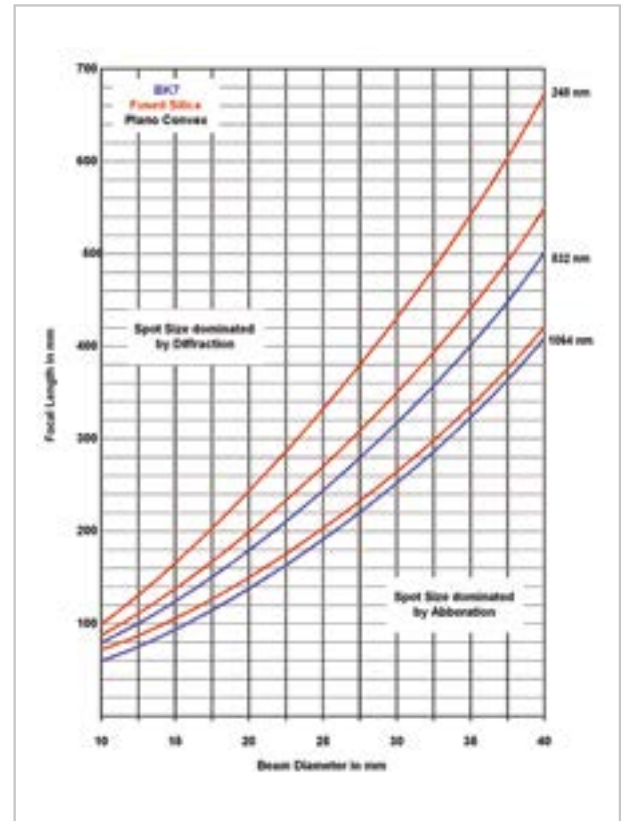
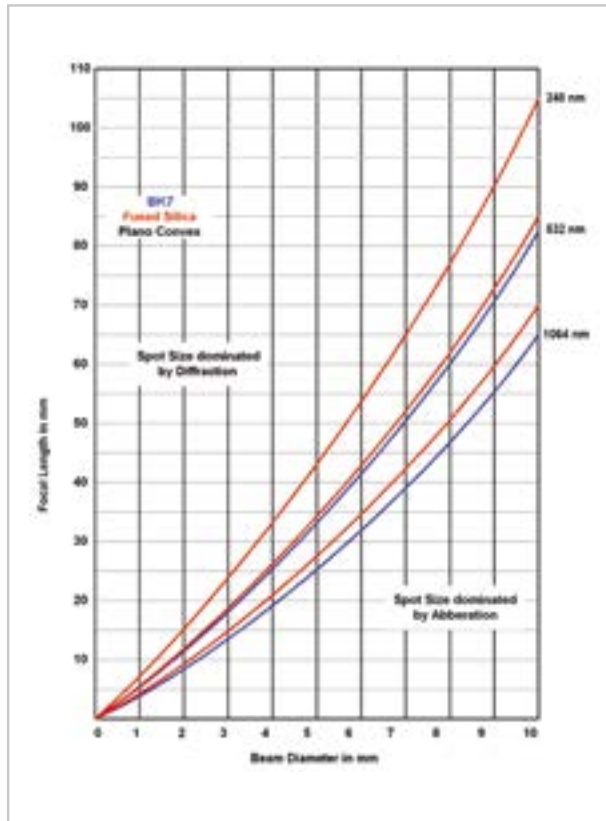
where if:

- $f \Delta\theta_{\text{blur}} > f \Delta\theta_{\text{diff}}$ the spot size is limited by aberration
- $f \Delta\theta_{\text{blur}} < f \Delta\theta_{\text{diff}}$ the spot size is essentially diffraction limited

The curves on the following page depict this computation for properly oriented plano-convex BK7 and quartz singlets.

Diffraction Limit

To use the curves, find the beam diameter and desired focal length point on the graph. If this point is above the curve for the intended wavelength, plano-convex singlet lens performance is essentially diffraction limited. If this point is on or below the curve, consider using a doublet to decrease the aberrated spot size.



Note

In designing optical systems for low divergence laser beams, the paraxial approximation is excellent as far as the positioning of the elements is concerned. However, due to diffraction, the magnifications encountered may be completely different than those calculated from geometrical optics.

Note the following method:

- Use the familiar ABCD matrix formalism.
- Treat the lenses as thin lenses.
- Use the distances between the principal points to determine the element positioning.
- If you use the actual radii of curvature, refraction indices, and center thicknesses in a more detailed calculation, your calculation will already implicitly take into account the principal planes.